

Reconstruction of the Thermal Conductivity Depth Profile of Hardened Steel Rods

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In the first part of this work, we calculate the surface temperature of (a) a multilayered cylinder and (b) a solid cylinder whose in-depth thermal conductivity vary continuously when they are illuminated by a modulated light beam. It is demonstrated numerically that the temperature behavior corresponding to the continuously varying thermal conductivity is well reproduced by using a small number of layers, provided their thicknesses are not constant, but thinner as they approach to the surface. Accordingly, we have used a 20-layers-model to reconstruct the whole thermal conductivity depth profile of hardened C1018 steel rods. The recovery of the internal conductivity from measurements of the surface temperature is a severely *ill-posed* non-linear inverse problem; i.e. the unknown conductivity is extremely sensitive to errors in the photothermal signal. In order to retrieve stable inversions, we have applied the Tikhonov regularization method for the non-linear inverse problem. The agreement between the experimental data obtained by photothermal radiometry and the theoretical fits is very good in the whole frequency range of the experiments (0.5 Hz to 10 kHz). The reconstructed thermal conductivity depth profile has a similar shape to those found for flat samples. Moreover, it shows qualitative anticorrelation with the hardness depth profile, which has been obtained from mechanical indentation tests. This work provides the mathematical tools for applying photothermal radiometry to retrieve the hardening depth profile of nails, tubes, and other metallic pieces of cylindrical shape in a nondestructive way.

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