Helium-4 State Equation from 0 to (Tlambda+0.001) K

Vincent Arp [C, S]
Cryodata, Inc., Boulder, CO, U.S.A.

Previous study has shown that the He^4 state equation below 0.8 K is made up of three classes of terms: (1) temperature independent compressibility terms, (2) Debye phonon excitations, and (3) roton excitations. The minimum roton energy is greater than that of the superfluid ground state by an amount E_g. In this work we seek additional or modified terms that extend He^4 equation through the lambda line. Near the Lambda line, it is convenient to define functions $L_2 = (A/a)\ |x|^{-a}$, where x is the isobaric distance, = $T-T_l(P)$, from the lambda line and a is constant. The only difference between superfluid state properties (x<0) and He I properties (x>0) is that the constant A is larger in He I than in He II by a universal factor customarily denoted "A/A'". Then defining L_1 and L_0 as successive integrals of L_2 (x), it is possible to create a general Gibbs potential $G(P,T) = A_L(P,T)\ L_0(x) + B(P,T)$ which forms the state equation near the lambda line. Following the same reasoning but along an isochore, it is possible to create a Helmholtz potential function $A(r,T) = A_L(r,T)\ L_0(x) + B(r,T)$. With careful selection of polonomial or other functions for the A_L and B terms, the valid range for these equations can be extended somewhat beyond the immediate vicinity of the Lambda line. Previous He^4 state equations have implicitly used equations of this form, but it seems extremely awkward to extend such equations into the phonon region below about 1 K. The new feature in this work is to recognize the function L_0 not as a potential function per se, but rather as the energy gap E_g in the roton excitation equation. Integration constants in the L_1 and L_0 functions provide an array of adjustable parameters. Theoretical valuations on a and A/A' are included. The final result is that we fit state properties from 0 to (T_l+0.001 K) with an equation of 24 linear terms. Supporting reference data were obtained primarily from NIST archives. About one-third of the fitted data were within 10^{-3}K of the lambda line. Special effort has been made to obtain a satisfactory fit near the intersection of the lambda line with the melting line, which point exhibits the most resistance to fitting function accuracy. Overall accuracy of state properties equals and generally exceeds that of previous equations valid in more restricted ranges of temperature and pressure.